Int. J. Solids Structures, 1969, Vol. 5, pp. 941 to 950. Pergamon Press. Printed in Great Britain

FAILURE OF NOTCHED COLUMNS WITH FIXED ENDS

H. LIEBOWITZ

Office of Naval Research, and The George Washington University, Washington, D.C.

and

W. D. CLAUS, JR.

Princeton University, and Textile Research Institute, Princeton, New Jersey

Abstract—A theoretical basis is given for some of the recent experimental results [4] for the maximum load-carrying capacity of sharply notched aluminum 7075-T6511 columns with fixed-ends. A failure criterion based on the stress–intensity factor concept is developed for these concentrically loaded, notched columns. The criterion is used to predict failure loads which are found to be in good agreement with experimental results for the case of fixed-end, notched aluminum columns investigated in this study. This work is another example of how the fracture mechanics concept may be applied to determining the response of notched columns under axial compression loading.

INTRODUCTION

STRUCTURAL engineers are constantly faced with the problem of determining the maximum loads allowable for a particular structure. Design practices are revised often to embrace modern research results. The recent program of Liebowitz *et al.* [1–3] includes a systematic investigation of the effect of a particular type of flaw, namely a sharp notch, on the load carrying capacity of structural columns. The experimental and analytical work in [1–3] pertain to notched aluminum columns with pinned-ends. In this paper, a failure criterion is formulated for notched columns with fixed-ends, and the theoretical results are compared with the experimentally determined failure loads given in Ref. [4].

The experimental studies mentioned above [4] were compression tests carried out on notched and unnotched aluminum columns with square cross-sections. The columns were loaded nearly concentrically and the maximum carrying load reached was measured. Various length columns with single notches on one side and double notches on opposite sides of varying notch depths were studied. A detailed description of the experimental apparatus and results was given in Ref. [4]. For later use, it is noted here that the test specimens were made from 7075-T6511 aluminum bars. The pertinent material properties are: Young's modulus of 10^7 psi, yield stress of 73,000 psi, ultimate stress of 83,000 psi, and a stress–intensity factor (K factor) of 38,000 lb-in$^{-\frac{3}{2}}$.

FAILURE CRITERION

The objective of this section is to formulate a failure criterion for notched columns with fixed-ends. The procedure followed is an extension of the criterion developed in Ref. [3] which was based on the stress-intensity (K) factor concept. The K factor may be defined by

$$K = \frac{(\pi)^{\frac{1}{2}}}{2} \lim_{\rho \to 0} (\rho)^{\frac{1}{2}} \sigma_{\max}$$

where σ_{max} is the maximum stress at the notch root and ρ is the notch radius. Using Neuber's results for stress distributions in hyperbolic notches, a combined K factor accounting for both bending and compressive stresses in a beam of rectangular cross-section was derived in Ref. [3] and is

$$K = 2\left(\frac{b}{\pi}\right)^{\frac{1}{2}}[\tfrac{2}{3}(\sigma_{nom})\text{ bending} + (\sigma_{nom})\text{ compression}] \tag{1}$$

where the nominal stresses are evaluated at the notch root. The K factor is valid for double, symmetric hyperbolic notches with a throat width of $2b$.

Since the nominal stresses are functions of the applied load P and K is a material property, equation (1) is interpreted as a failure criterion giving the maximum carrying load. Furthermore, the boundary conditions of the column are taken into account through the selection of the nominal stresses. In Ref. [3] pinned-end columns were considered: in this paper, fixed-end columns are investigated.

It is known that if a perfectly concentric load were applied to a perfectly straight column, buckling, not bending, would occur. To take into account the observed bending, an equivalent initial crookedness of the column

$$y_0(x) = \delta \sin\frac{\pi x}{L}$$

is assumed. Using standard, classical techniques [5] the bending stress at the middle and at the outer fiber of a fixed-end column of width $2b$ under an axial load $P$is

$$\sigma_b = \frac{P\delta b}{I(1-\alpha)}\left[1-\frac{\pi\alpha}{2u\sin u}\right] \tag{2}$$

where

$$\alpha = PL^2/(\pi^2EI) \quad\Big|\quad u = (L/2)(P/EI)^{\frac{1}{2}} \quad\Big|\quad P_{cr} = 4\pi^2EIL^{-2}$$
$$\alpha = 4P/P_{cr} \quad\Big|\quad u = \pi(P/P_{cr})^{\frac{1}{2}}$$

This stress is taken as the nominal bending stress in equation (1). The nominal compressive stress is taken to be $-P(2bh)^{-1}$. Here h is the thickness of the bar.

Substituting the nominal bending stress equation (2) and the nominal compressive stress into equation (1) allows rewriting the K factor as

$$K = A\left[1-\frac{2(P/P_{cr})^{\frac{1}{2}}}{\sin[\pi(P/P_{cr})^{\frac{1}{2}}]}\right] - P(\pi b)^{-\frac{1}{2}}h^{-1}$$
$$A = 2P\delta\pi^{-\frac{1}{2}}h^{-1}b^{-\frac{3}{2}}[1-4(P/P_{cr})]^{-1} \tag{3}$$

This relation is regarded as a failure criterion for the notched columns with fixed-ends. K is an experimentally determined material property so that equation (3) can be solved numerically for the failure load P. Of course equation (3) is valid only for $P < P_{cr}$.

Several comments are in order regarding equation (3). First, the moment of inertia in the bending stress equation (2) was taken to be the reduced cross-sectional moment $2hb^3/3$ since the nominal stress at the notch root was the nominal stress desired. Second, it was found experimentally that the notch did not affect the buckling (P_{cr}) load of the straightened column. Hence P_{cr} was calculated using the full cross-section.

COMPARISON WITH EXPERIMENT

The failure criterion equation (3) was solved numerically for the failure load P for columns of various lengths with notches of various depths. The calculated failure loads were then compared with the experimental results reported in Ref. [4]. A summary of these results is presented in Figs. 1–12. In Figs. 1–4 the maximum load is plotted against total notch depth for columns 10·25, 16·25, 22·25 and 28·25 inches long. Experimental points are shown for both single and double notches for comparison. Although the single notch data seem to agree well with the calculated values of the maximum load, remember that the assumptions used in deriving the failure criterion restrict the validity of that criterion to the symmetric, double notch case. It is simply noted that the same criterion appears to be successful in the asymmetric, single notch case also. Further research needs to be done to clarify this point.

In Figs. 5–12 the maximum load is plotted against column length for various notch depths. Calculations were done using notch depths from 0·020 in. to 0·230 in. It is seen from these figures that agreement between calculated and measured maximum loads is acceptably good. An initial eccentricity of 0·1 in. gives good agreement with experimental results.

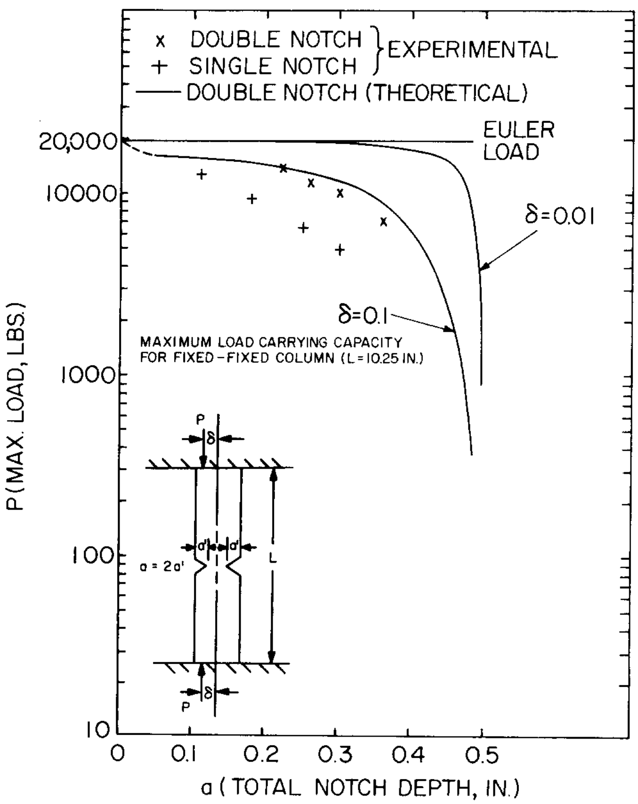

FIG. 1. Maximum load carrying capacity vs. total notch depth for fixed-fixed column (L = 10·25 in.).

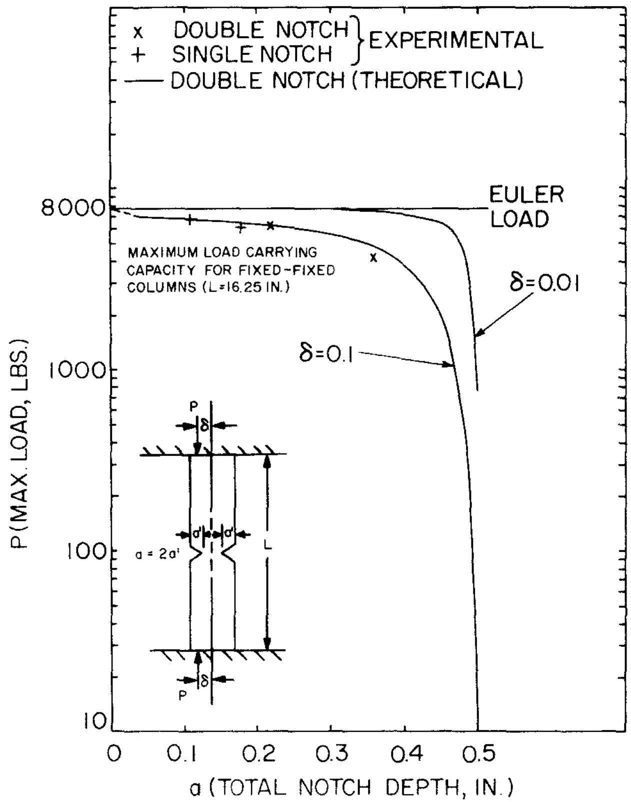

FIG. 2. Maximum load carrying capacity vs. total notch depth for fixed-fixed columns (L = 16·25 in.).

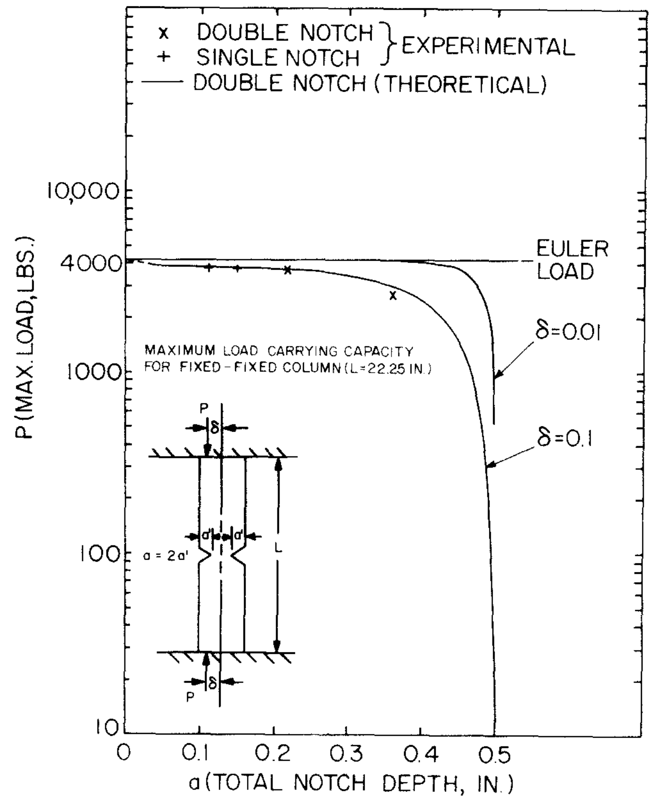

FIG. 3. Maximum load carrying capacity vs. total notch depth for fixed-fixed column (L = 22·25 in.).

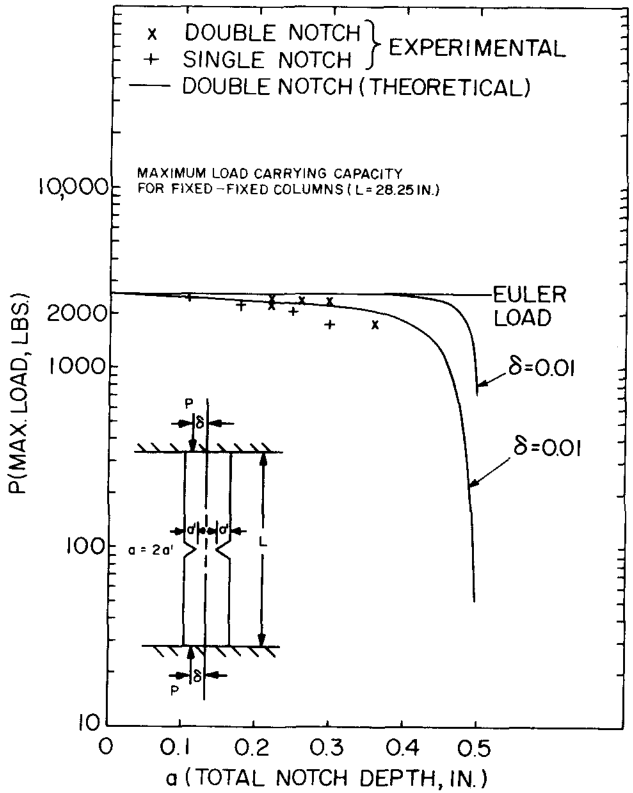

Fig. 4. Maximum load carrying capacity vs. total notch depth for fixed-fixed columns (L = 28·25 in.).

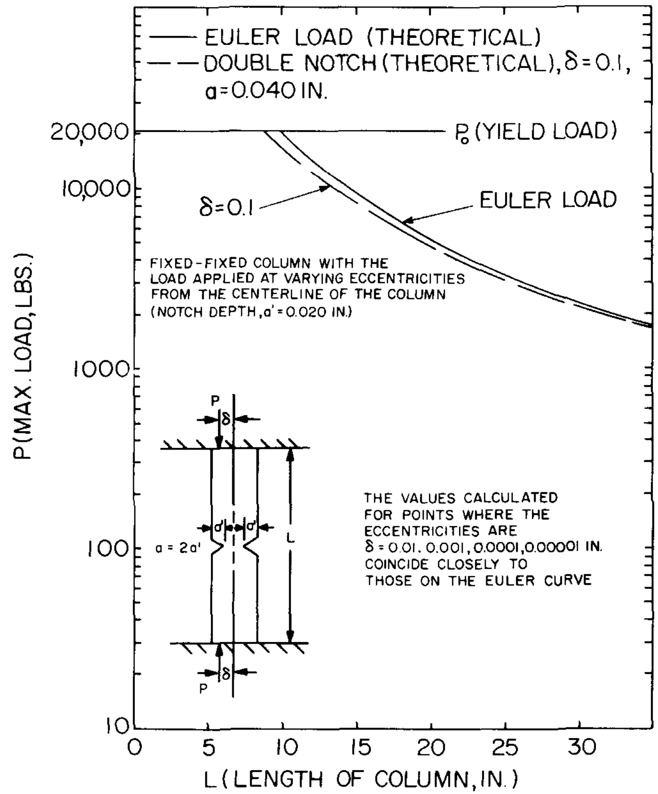

Fig. 5. Fixed-fixed column with the load applied at varying eccentricities from the centerline of the column vs. length of column (notch depth, a' = 0·020 in.).

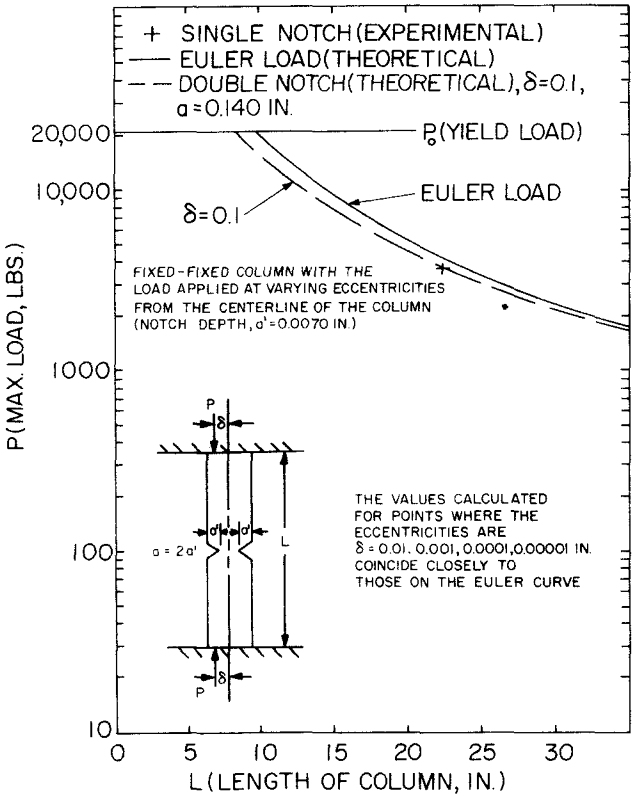

FIG. 6. Fixed-fixed column with the load applied at varying eccentricities from the centerline of the column vs. length of column (notch depth, $a' = 0{\cdot}0070$ in.).

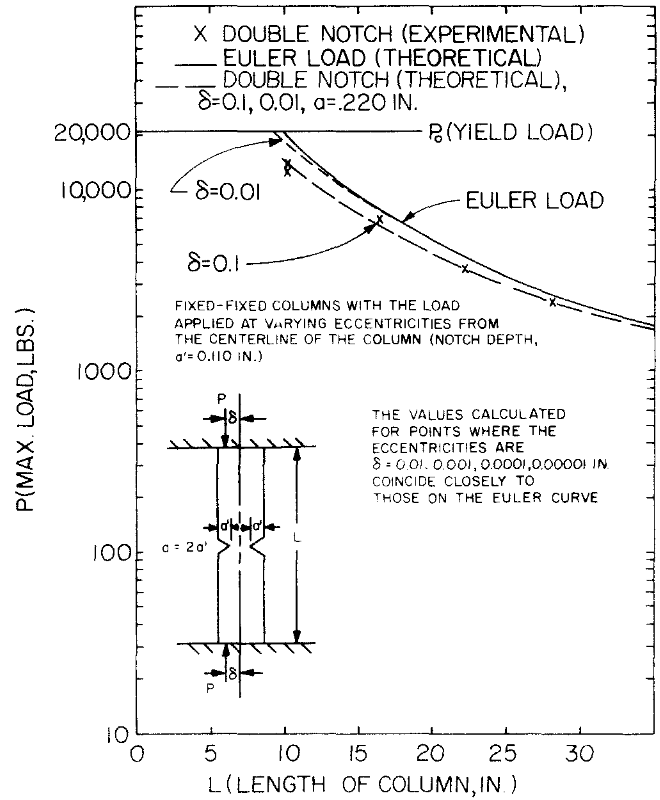

FIG. 7. Fixed-fixed columns with the load applied at varying eccentricities from the centerline of the column vs. length of column (notch depth, $a' = 0{\cdot}110$ in.).

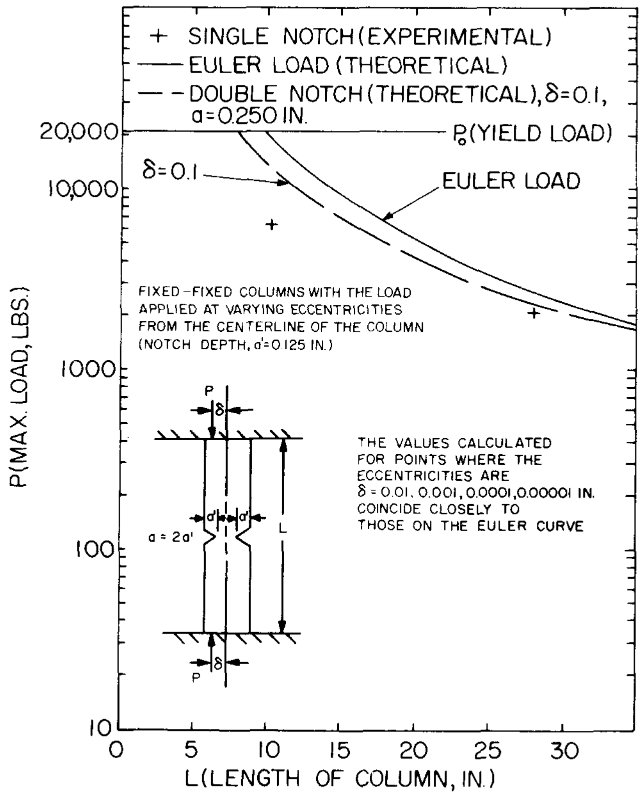

FIG. 8. Fixed-fixed columns with the load applied at varying eccentricities from the centerline of the column vs. length of column (notch depth, $a' = 0{\cdot}125$ in.).

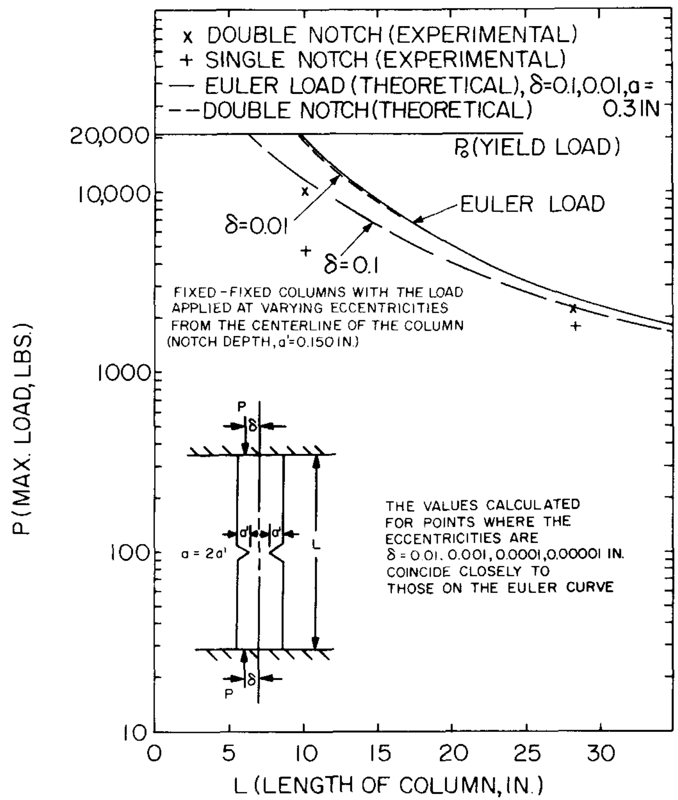

FIG. 9. Fixed-fixed columns with the load applied at varying eccentricities from the centerline of the column vs. length of column (notch depth, $a' = 0{\cdot}150$ in.).

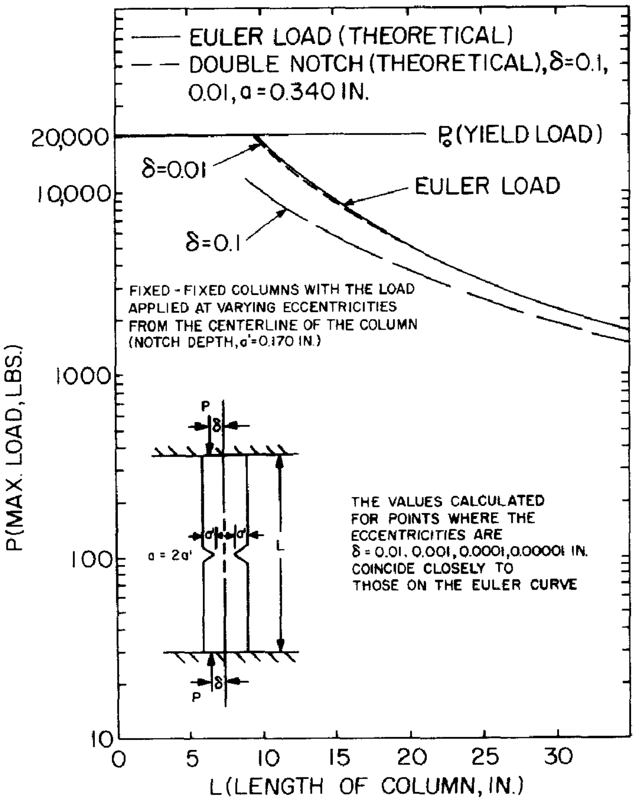

FIG. 10. Fixed-fixed columns with the load applied at varying eccentricities from the centerline of the column vs. length of column (notch depth, $a' = 0{\cdot}170$ in.).

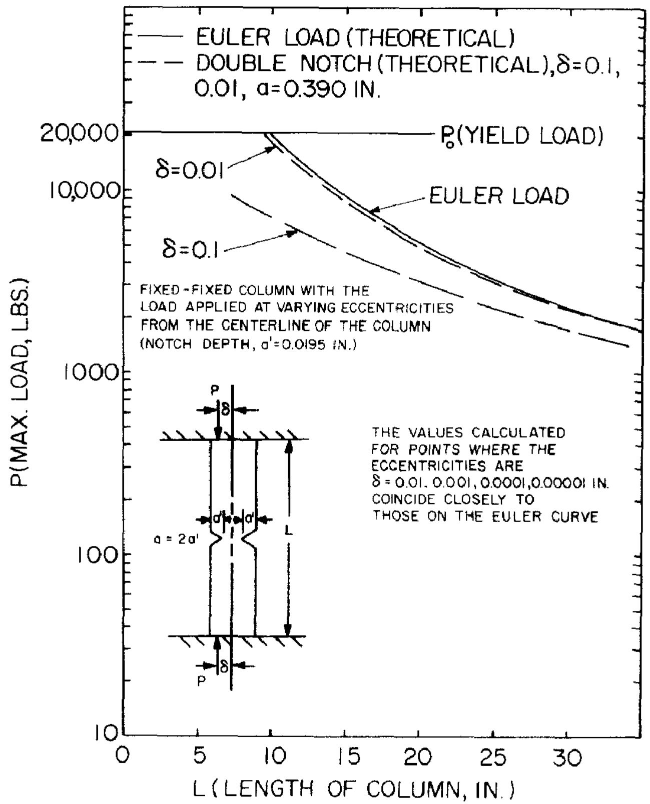

FIG. 11. Fixed-fixed column with the load applied at varying eccentricities from the centerline of the column vs. length of column (notch depth, $a' = 0{\cdot}0195$ in.).

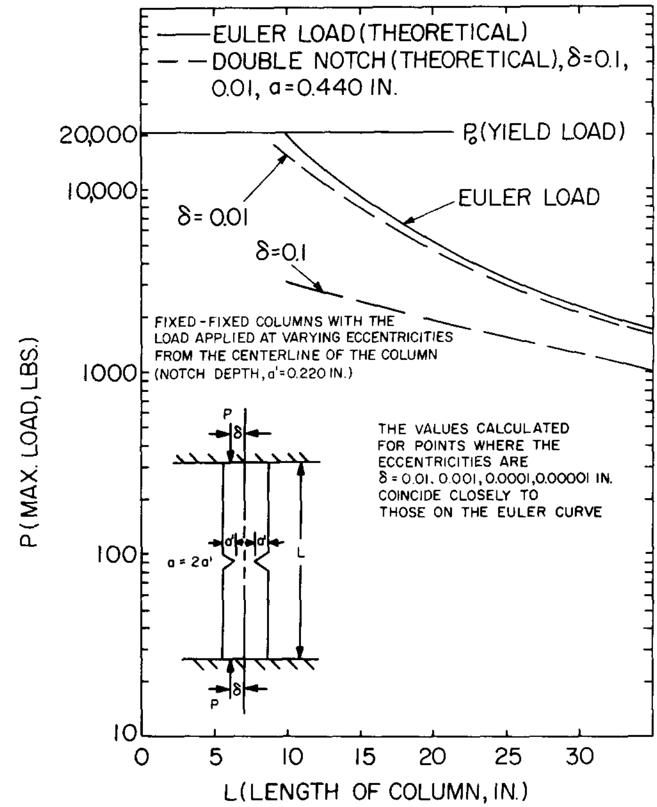

FIG. 12. Fixed-fixed columns with the load applied at varying eccentricities from the centerline of the column vs. length of column (notch depth, $a' = 0{\cdot}220$ in.).

CONCLUSIONS

The failure criterion developed in this paper equation (3), based on the stress intensity concept combined with Neuber's notch stresses, adequately predicts the failure loads of the notched, aluminum columns with fixed ends considered in this study. The criterion was successfully applied to $\frac{1}{2}$ in. by $\frac{1}{2}$ in. columns from 10·25 to 28·25 in. long with notches from 0·055 to 0·180 in. deep.

Acknowledgement—The authors thank the Catholic University of America, National Science Foundation, The George Washington University, and The Reynolds Metals Co., Inc. for their assistance in performing this investigation. Also, acknowledgement is made to Mr. H. Vanderveldt for making available experimental data and some pertinent calculations for this study.

REFERENCES

[1] H. Liebowitz, H. Vanderveldt and D. W. Harris, Carrying capacity of notched columns. *Int. J. Solids Struct.* **3**, 489–500 (1967).

[2] H. Liebowitz, D. W. Harris and H. Vanderveldt, Fracture of long and short notched columns, *Exp. Mech.* **8**, 49–56 (1968).

[3] H. LIEBOWITZ and W. D. CLAUS, JR., Failure of notched columns. *J. Engng fracture Mech.* 1, 379–385 (1968).
[4] H. VANDERVELDT and H. LIEBOWITZ, Failure of notched columns. *J. Engng fracture Mech.* in press.
[5] STEPHEN P. TIMOSHENKO and JAMES M. GERE, *Theory of Elastic Stability*, 2nd edition, p. 37. McGraw-Hill (1961).

(*Received* 27 *June* 1968; *revised* 2 *December* 1968)

Абстракт—Дается теоретическая основа некоторых последних экспериментальных результатов [4], касающихся максималъной несущей способности остро насеченных колонн, изготовленных из алюминия 7075–T6511, с защемленными концами. Рассматривается критерий разрушения, этих концентрически нагруженных колонн с надрезом, основанный на концепции фактора интенсивности напряжений. Этот критерий используется для определения нагрузок разрушения, хорошо согласующихся с экспериментальными результатами для случая защемленных, алюминиевых колонн с надрезом и с защемленными концами, исследуемых в настоящей работе. Предлагаемая работа дает еще один пример применения концепции механики разрушения для определния реакции колонн с надрезом, сжатых осевой нагрузкой.